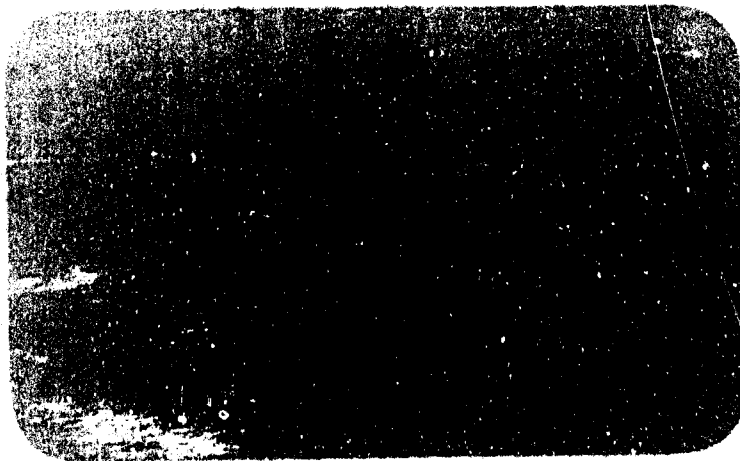AFCRL-65-102

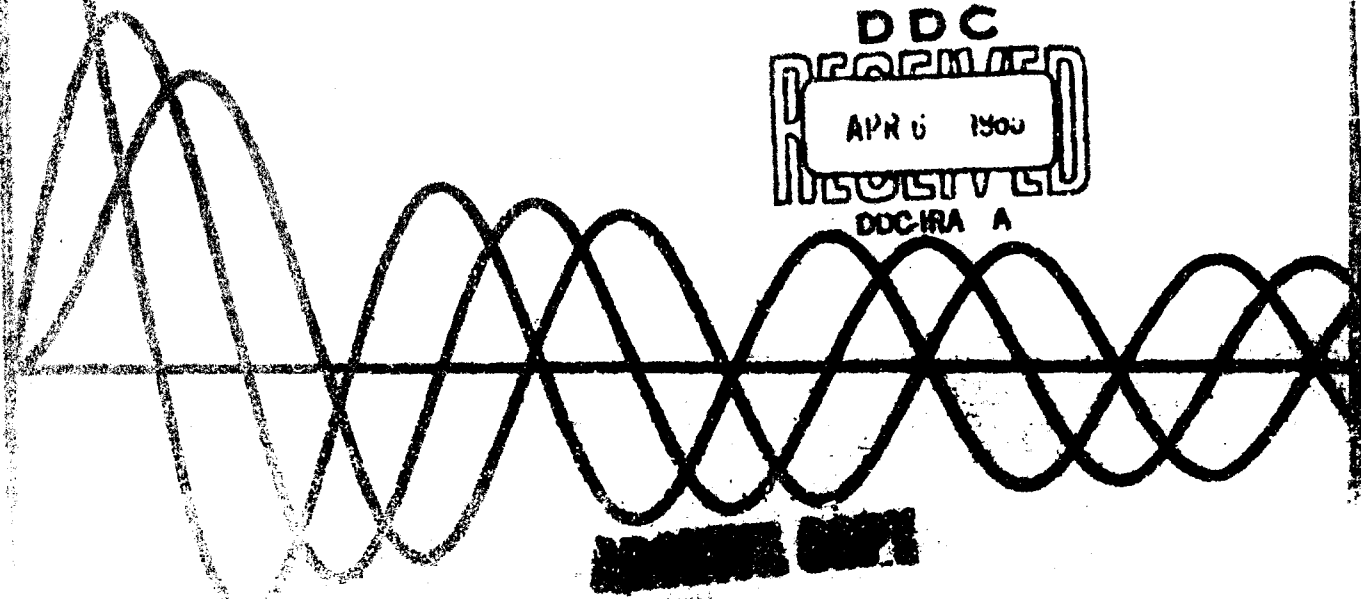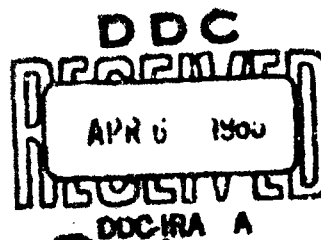*Laboratory* Best Available Copy
of

ELECTROMAGNETIC THEORY

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LYNGBY

A theoretical investigation

of Van Atta arrays

Tove Larsen

ABSTRACT

A scheme has been set up for an analytical investigation of an arbitrary Van Atta reflector. Each pair of antenna elements is represented by an equivalent circuit using equivalent x-circuits for the transmissionlines. Reradiation from the elements and mutual impedances have been taken into account. The theory is illustrated with some numerical examples of a linear reflector consisting of four half-wave dipoles.

TABLE OF CONTENTS

1. INTRODUCTION

In 1959 L.C. Van Atta[1] patented a new type of reflector, the advantage of which was that the reradiated field showed a maximum back in the direction of arrival of the primary plane wave. Some experimental investigations were carried out on this type of reflector and many suggestions for improvement and utilizations of the reflector were given. However the development has run so fast, that a thorough theoretical treatment of the properties of the basic Van Atta reflector never was given (at least such one has not come to the knowledge of the present author). At this laboratory a theoretical and numerical investigation[2] of linear Van Atta reflectors was carried out in 1963 as a M.Sc. thesis work. These calculations showed that a Van Atta reflector only to some degree will behave as predicted in the patent description. It is the purpose of this note to set up a general scheme for a theoretical treatment of a Van Atta array and to illustrate the theory with some numerical results obtained by using the previous computer program[2] in a slightly modified version.

2. THE VAN ATTA REFLECTOR

The basic idea of the Van Atta reflector as given in the patent description and as repeated by all subsequent authors is illustrated in fig. 1 for a one dimensional array. The reflector consists of a number of antenna elements, which are mutually interconnected with transmissionlines of equal length. If the reflector consists of an odd number of elements, the center one is connected to a short-circuit transmission line with half the length of the other lines. A plane wave approaching this array with the angle of incidence θ_i will induce voltages in the elements the phase of which will grow along the array with an amount kd sin θ_i, where k is the propagation constant and d the distance between adjacent elements(shown in twist line of fig. 1). These induced signals will be transmitted through the connecting lines and will produce a current distribution with the relative phases as shown in the bottom line of fig. 1. It is seen that a current distribution like this will produce a plane wave reradiated back in the direction of arrival of the primary wave.

Two facts are neglected in this simple explanation of the way of action of the reflector. First it is assumed that the elements do not reradiate any amount of the energy induced in themselves, and secondly mutual impedances are neglected. If these facts are taken into account both the length of the transmissionlines and the distance between adjacent elements will influence the properties of the reflector. In section 5 it will be shown that if the two facts mentioned are taken into account the reradiation pattern will take various forms depending on the parameters of the reflector. On the other hand it is shown in the appendix, that if the two above mentioned facts are neglected, the reradiation pattern will be as predicted in the patent description.

3. SURVEY OF PREVIOUS WORK

L.C. Van Atta's patent was applied for on June 8, 1955 and granted October 6, 1959. The idea of the reflector was described in the literature already in 1956 by Bloch[3], but the main bulk of literature appeared after Van Atta had got his patent in 1959. Most of the papers have concerned the use of auxiliary active equipment in connection with a Van Atta reflector or the various applications of this type of reflector. Only the experimental work of Sharp, Fusca and Diab [4)5)6) on an electromagnetic reflector and that of Walther [7] on an acustic reflector seems to be of basic character. No thorough theoretical treatment of the properties of a simple Van Atta reflector has been found in the literature.

The work of Sharp, Fusca and Diab describes the construction and examination of a two dimensioned array consisting of 16 dipoles mounted on quarter of a wavelength above a conducting plane. A simplified theory for this array was given and a number of diagrams of the back scattering cross section as a function of the angle of incidence were presented. These diagrams were measured for various polarisations and frequencies. For all the examples shown the array had a high back-scattering cross section over a wider range of angles of incidence than the normal flat plate reflector of the same size.

Walther constructed an acoustic Van Atta array consisting of 36 conical horns arranged on a flat surface. Measured back scattering cross section diagrams showed high reflection over a greater range of angles of incidence that a conventional reflector of the same size of aperture.

Two short notes tend to give an analytical treatment of the Van Atta reflector. Bauer[8] has as the only one taken into account reradiation from the antenna elements themselves, but as the main purpose of his note is to suggest amplitude modulation of the reflector, he does not mention the influence of other parameters on the properties of the basic reflector. Furthermore his expression (6) for the total reradiated field is wrong, as he has forgotten the phase difference due to propagation from the antennas to the wavefront of the reflected wave. Kuress and Kahn[9] give a short general theory for the effect of lossless interconnections of elements in a passive array. However they anticipate without justification that the current distribution causing reradiation show the same numerical value on all elements and a uniform phase progression, which should cause a reradiated plane wave directed either back in the direction of the incoming wave (Van Atta principle) or in a direction determined by an angle of refelction equal to the angle of incidence with opposite sign (normal plane reflector principle).

The idea of making the Van Atta reflector active by inserting active components in the transmission lines was first given by Bauer[8], who suggested the introduction of modulated phase shifters in the transmission lines. The insertion of amplifiers in the transmission line was suggested by Hansen[10]. Mechanical modulation by means of a cavity resonator was proposed by Wanselon[11]. Davies[12] discussed the effect of amplifiers in the transmission lines, as well as the features of a circular Van Atta array. Further he proposed a way of varying the angle of retransmission by introducing a frequency change in the delay paths. A duscussion[13] of Davies' work was later on published together with the description of an experimental investigation related to Davies' theories and carried out by Whithers[14].

One application of the Van Atta reflector is for satellite communication, and both passive, semipassive and active systems have been discussed, especially by Ryerson[15,16], Hansen[10] and Kaiser and Kay[17]. Other application are for navigational aids, for example used to enhance the reflection from radartargets on small ships and airplanes (discussed by Davies[12]). Fusca[5] suggested an ECM system based on the idea that and artificial enhancement of radar returnsignals would confuse enemies. Bauer[8] suggested sort of a passive IFF system by using a coded modulation of the reflector. Bauer's idea was criticised by Bahret[18].

The Van Atta reflector is considered to be one of the most simple forms of the group of adaptive or selfphasing arrays, which has received a great deal of interest in the recent literature[19,20]. Most of these systems are active and rather complicated, and the applications seem to be numerous.

4. THEORETICAL INVESTIGATION

In what follows an analysis of the properties of a Van Atta reflector will be given taking into account all relevant characteristics. The aim has been to give a general treatment, which could be used for a number of investigations. The elements are supposed to be dipoles (but the theory could easily be extended to other antenna types) placed on and parallel to a (imaginary) smooth surface, for example a plane, a cylinder, a sphere. The incident field is a plane wave.

The courses of the calculations are as follows. First the open circuit voltage induced at each antenna element by the primary plane wave is calculated (sec. 4.1). Next a system of equations are set up (sec. 4.4) for calculating the currents in each antenna taking into account mutual impedances (sec. 4.2), the characteristics of the transmission lines (sec 4.3) and the induced voltage at the element itself and at its mate. When the currents are determined the reradiated field is calculated from the theory of antenna arrays (only applicable for reflectors with all elements parallel) (sec. 4.5). Finally the properties of the reflector array are determined by the calculation of the differential scattering cross section (sec. 4.6).

4.1. The induced voltage

In fig. 2 is shown the reflector surface and the incident plane wave. A reference plane is introduced as the plane tangential to the surface through a conveniently selected point O. Further a rectangular coordinate system with O as the center and the Z-direction perpendicular to the plane is introduced ($\hat{x}$, $\hat{y}$ and $\hat{z}$ denote unit vectors). The angles of incidence of the primary plane wave in this system is ϕ_i and Θ_i, and the angle between the electric vector $\vec{E}_0$ of the incident wave and the plane of incidence is v. A dipole antenna element is placed at the point (X_n, Y_n, Z_n) inclined the angles ϕ_n and Θ_n to the X- and Z- axes respectively.

Now the open-circuit voltage induced at the terminals of an antenna element will be given by

$$V = f\vec{E}_0 \cdot \vec{L}_{eff} ,\qquad (1)$$

where f is a phase factor and $\vec{L}_{eff}$ the effective length[21] of the antenna element.

If the phase of the plane wave in the plane through O is zero we have (time factor $e^{-i\omega t}$)

$$f = e^{-ik(X_n \cos\phi_i \sin\theta_i + y_n \sin\phi_i \sin\theta_i + Z_n \cos\theta_i)} . \tag{2}$$

In the reference coordinate system we have for $\bar{E}_o$:

$$\bar{E}_o = E_o \left[(-\sin v \sin\phi_i - \cos v \cos\phi_i \cos\theta_i) \, \hat{x} \right.$$
$$\left. + (\sin v \cos\phi_i - \cos v \sin\phi_i \cos\theta_i) \, \hat{y} + \cos v \sin\theta_i \, \hat{z} \right] . \tag{3}$$

The quality $\bar{L}_{eff}$ has the same direction as the antenna element and will for dipole be given by

$$\bar{L}_{eff} = \frac{\lambda}{\pi} \frac{\cos (\frac{\lambda}{\pi} L \cos u) - \cos \frac{\pi}{\lambda} L}{\sin \frac{\pi}{\lambda} L \sin u} (\sin\theta_n \cos\phi_n \, \hat{x}$$
$$+ \sin\theta_n \sin\phi_n \, \hat{y} + \cos\theta_n \, \hat{z}) \quad , \tag{4}$$

where L is the length of the dipole, and u is the angle between the direction of the dipole and the direction of propagation of the primary plane wave. With our notation we have that u is given by

$$\cos u = \sin\theta_n \sin\theta_i \cos(\phi_n - \phi_i) + \cos\theta_n \cos\theta_i \quad . \tag{5}$$

It is seen that the induced voltage may be written

$$V = E_o \frac{\lambda}{\pi} f(P_n, \theta_i, \phi_i) \, p \, (\theta_i, \phi_i, v, \theta_n, \phi_n, L) \quad , \tag{6}$$

where f is the complex phase factor with numerical value one given in (2), and p is the real dimensionless quantity

$$p = \frac{\cos (\frac{\pi}{\lambda} L \cos u) - \cos \frac{\pi}{\lambda} L}{\sin \frac{\pi}{\lambda} L \sin u} \times$$
$$(\sin\theta_n \cos\phi_n \, (- \sin v \sin\phi_i - \cos v \cos\phi_i \cos\theta_i)$$
$$+ \sin\theta_n \sin\phi_n \, (\sin v \cos\phi_i - \cos v \sin\phi_i \cos\theta_i)$$
$$+ \cos\theta_n \cos v \sin\theta_i) \quad . \tag{7}$$

4.2. Self- and mutual impedances

The problems involved in determining the self- and mutual impedances of antennas are discussed in most textbooks on antenn theory, f.ex. Jordan's book 22).

Using the induced-EMF method the self-impedance Z_A of a dipole antenna of total length L (fig. 3 a) is given by

$$Z_A = - \int_0^L \frac{I(z) E_z(z)}{I_j^2} \, dz \quad , \tag{8}$$

where I_1 is the terminal current, $I(z)$ the current in the antenna, and $E_z(z)$ the electrical field strength at and parallel to the antenna due to the own field.

The mutual impedance between two antennas is defined as the ratio between the open-circuit voltage induced across the terminals of one antenna due to a current flowing in the other antenna, and this current. Using the induced-EMF method the mutual impedance between two dipoles (fig. 3 b) is given by

$$Z_{12} = - \int_0^{L_2} \frac{I_2(z)\, E_{z21}(z)}{I_1 I_2}\, dz \; ,$$

$$= - \int_0^{L_1} \frac{I_1(\alpha)\, E_{\alpha12}(\alpha)}{I_1 I_2}\, d\alpha \; , \tag{9}$$

where $I_2(z)$ is the current in antennas 2, $I_1(\alpha)$ the current in antenna 1, $E_{z21}(z)$ the eltrical field strength at an parallel to antenna 2 due to a current in antenna 1, and $E_{\alpha12}(\alpha)$ the field strength at and parallel to antenna 1 due a current in antenna 2. I_1 and I_2 are the terminal currents of antennas 1 and 2, respectively.

These integral expressions for the impedance will usually lead to very lengthy and complicated computations, which have been carried out in special cases only. Stearns[23] has issued a table of the mutual impedances between parallel side-by-side, half-wave dipoles with sinusoidal current distribution. This table has been used in the numerical examples discussed in section 5. At this laboratory two Algol-computer programs[24] for the computation of mutual impedances between antennas have been worked out. One is for linear dipoles of arbitrary length and direction sinusoidal current distribution, the other for completely arbitrary wire-antennas with a known current distribution.

4.3. Equivalent diagram for a transmission line

A section of length a of a transmission line may as other passive two-ports be represented by an equivalent circuit. Usually a T-circuit or a π - circuit is used, but in order to get an equivalent circuit, which is valid for all values of a it is necessary to use the more general x-circuit. This circuit is shown in fig. 4 a. It is assumed to be symmetrical and lossless. The values of the impedances Z_M and Z_N are

$$Z_M = - i Z_0 \, tg \, \frac{ka}{2} \; . \tag{10}$$

$$Z_N = iZ_o \frac{1}{tg\frac{ka}{2}} \quad , \tag{11}$$

where Z_o is the characteristic impedance of the transmission line.

The variation of these impedances with a is shown in fig. 4 b. It is seen that for $a = (2p + 1)\lambda$, Z_M is zero and Z_N infinite. This gives the simple equivalent circuits shown in fig. 4 c and known from ordinary transmission line theory.

4.4. Equations for determination of antenna currents

Let us consider a reflector array consisting of N elements numbered n, $1 \leq n \leq N$. The elements are mutually interconnected with transmission lines, and if N is odd one of the elements is connected to a short-circuited transmission-line. In this way we get S pairs, numbered s, $1 \leq s \leq S$. For N even, $S = \frac{N}{2}$, and for N odd, $S = \frac{N-1}{2}$. Pair no. s consits of elements nos $n = s$ and $n = (N+1-s)$ connected with a transmission line. For N odd we get an additional system, numbered $s = S+1$, consisting of element no. $n = \frac{N+1}{2}$ connected to a short-circuit transmission line.

The self-impedance of each of the elements is called Z_A, and the mutual impedance between elements nos. m and k $Z_{mk} = Z_{km}$. The open-circuit voltage induced by the primary plane wave at element no. m is V_m, and the total current in the same element is I_m.

In fig. 5 a is shown the equivalent circuit of a pair of antenna elements, and in fig. 5 b is shown the equivalent circuit of an element connected to a short circuited transmission line. For the sake of simplicity the elements are numbered m, k and p respectively (according to the definitions given above $k = N+1-m$, $p = \frac{N+1}{2}$).

The mesh currents I' and I'' are introduced as shown in fig. 5 a. We have

$$I_m = I' + I'' \quad ,$$

$$I_k = -I' + I'' .$$

From the mesh described by I' we get

$$V_m - V_k = (Z_A + Z_N - Z_{mk})(I_m - I_k) + \sum_{\substack{n=1 \\ n\neq m,k}}^{N} I_n(Z_{nm} - Z_{nk}) \tag{12}$$

and from the mesh described by I'' we get

$$V_m + V_k = (Z_A + Z_N + Z_{mk})(I_m + I_k) + \sum_{\substack{n=1 \\ n\neq m,k}}^{N} I_n(Z_{nm} + Z_{nk}) . \tag{13}$$

From fig. 5 b we obtain directly

$$V_p = I_p(Z_A + 2 \frac{Z_N Z_M}{Z_N + Z_M}) + \sum_{\substack{n=1 \\ n \neq 1}}^{N} I_n Z_{np} . \tag{14}$$

Next we want to normalize these equations to include dimensionless quantities only. Looking at equations (6) section 4.1 for the induced voltage it is seen, that a suitable normalization factor for the voltage is

$$V_o = \frac{E_o \lambda}{\pi} . \tag{15}$$

A convenient normalization impedance would be the characteristic impedance Z_o of the transmission line. This yields the normalization current

$$I_o = \frac{E_o \lambda}{\pi Z_o} . \tag{16}$$

Further we introduce the values for Z_N and Z_M found in section 4.3. Using lower case for normalized quantities we finally get

$$(v_m - v_k) \cos \frac{ka}{2} = (- i \sin \frac{ka}{2} + (z_A - z_{mk}) \cos \frac{ka}{2})(i_m - i_k)$$
$$+ \sum_{\substack{n=1 \\ n \neq m,k}}^{N} i_n(z_{nm} - z_{nk}) \cos \frac{ka}{2} , \tag{17}$$

$$(v_m + v_k) \sin \frac{ka}{2} = (i \cos \frac{ka}{2} + (z_A + z_{mk}) \sin \frac{ka}{2})(i_m + i_k)$$
$$+ \sum_{\substack{n=1 \\ n \neq m,k}}^{N} i_n(z_{nm} + z_{nk}) \sin \frac{ka}{2} , \tag{18}$$

$$v_p \cos ka = i_p (z_A \cos ka - i \sin ka) + \sum_{\substack{n=1 \\ n \neq p}}^{N} i_n z_{np} \cos ka . \tag{19}$$

This way of writing has been introduced in order to avoid the infinite values of z_n and z_M. From the equations it is seen right away, that for

$$a = (2p + \lambda) \frac{1}{4} , \qquad i_p = 0 , \tag{20}$$

$$a = (2p + \lambda) \frac{1}{2} , \qquad i_m = i_k , \tag{21}$$

$$a = p\lambda \qquad , \qquad i_m = - i_k . \tag{22}$$

It is now possible to set up a matrix equation for the total number of systems:

$$
\begin{pmatrix}
\bar{v}_1 \\ \bar{v}_2 \\ \cdot \\ \cdot \\ \bar{v}_s \\ \cdot \\ \cdot \\ \bar{v}_S \\ (v^o_{S+1}) \\ v^+_1 \\ v^+_2 \\ \cdot \\ \cdot \\ \cdot \\ v^+_s \\ \cdot \\ \cdot \\ v^+_S
\end{pmatrix}
=
\begin{pmatrix}
\bar{z}_{11} & \bar{z}_{12} & \cdots & \bar{z}_{1n} & \cdots & \bar{z}_{1N} \\
\bar{z}_{21} & \bar{z}_{22} & \cdots & \bar{z}_{2n} & \cdots & \bar{z}_{2N} \\
\cdot & \cdot & & \cdot & & \cdot \\
\cdot & \cdot & & \cdot & & \cdot \\
\bar{z}_{s1} & \bar{z}_{s2} & \cdots & \bar{z}_{sn} & \cdots & \bar{z}_{sN} \\
\cdot & \cdot & & \cdot & & \cdot \\
\cdot & \cdot & & \cdot & & \cdot \\
\bar{z}_{S1} & \bar{z}_{S2} & \cdots & \bar{z}_{Sn} & \cdots & \bar{z}_{SN} \\
(z^o_{S+1,1}) & (z^o_{S+1,2}) & \cdots & (z^o_{S+1,n}) & \cdots & (z^o_{S+1,N}) \\
z^+_{11} & z^+_{12} & \cdots & z^+_{1n} & \cdots & z^+_{1N} \\
z^+_{21} & z^+_{22} & \cdots & z^+_{2n} & \cdots & z^+_{2N} \\
\cdot & \cdot & & \cdot & & \cdot \\
\cdot & \cdot & & \cdot & & \cdot \\
\cdot & \cdot & & \cdot & & \cdot \\
z^+_{s1} & z^+_{s2} & \cdots & z^+_{sn} & \cdots & z^+_{sN} \\
\cdot & \cdot & & \cdot & & \cdot \\
\cdot & \cdot & & \cdot & & \cdot \\
z^+_{S1} & z^+_{S2} & \cdots & z^+_{Sn} & \cdots & z^+_{SN}
\end{pmatrix}
\begin{pmatrix}
i_1 \\ i_2 \\ \cdot \\ \cdot \\ \cdot \\ i_n \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ i_N
\end{pmatrix}
\quad , (23)
$$

where

$$v^-_s = (v_s - v_{N+1-s}) \cos \frac{ka}{2} \quad , \tag{24}$$

$$v^+_s = (v_s + v_{N+1-s}) \sin \frac{ka}{2} \quad , \tag{25}$$

$$z^-_{ss} = - z^-_{s,N+1-s} = - i \sin \frac{ka}{2} + (z_A - z_{s,N+1-s}) \cos \frac{ka}{2} \quad , \tag{26}$$

$$z^-_{sn} = - z^-_{N+1-s,n} = (z_{ns} - z_{n,N+1-s}) \cos \frac{ka}{2} \, , \, n \neq s, N+1-s \, , \tag{27}$$

$$z^+_{ss} = z^+_{s,N+1-s} = i \cos \frac{ka}{2} + (z_A + z_{s,N+1-s}) \sin \frac{ka}{2} \quad , \tag{28}$$

$$z^+_{sn} = z^+_{N+1-s,n} = (z_{ns} + z_{n,N+1-s}) \sin \frac{ka}{2} \, ; \, n = s, \, N+1-s \quad . \tag{29}$$

For N odd we get the additional quantities (shown in brackets in (23))

$$v^o_S = v^o_{S+1} \cos ka \quad , \tag{30}$$

$$z^o_{S+1,S+1} = z_A \cos ka - i \sin ka \quad , \tag{31}$$

$$z^o_{S+1,n} = z_{S+1,n} \cos ka \quad ; \quad n \neq S+1 \quad . \tag{32}$$

All the quantities of the matrix equation (23) are complex. So in order to perform numerical computations a 2N-dimensional matrix equation has to be solved. A typical single equation will for N=2 have the form

$$v = z_1 i_1 + z_2 i_2$$

or with subscripts r and i denoting real and imaginary parts, respectively, and with r and x being real and imaginary parts of the impedances we get

$$v_r = r_1 i_{1r} - x_1 i_{1i} + r_2 i_{2r} - x_2 i_{2i} \; ,$$

$$v_i = x_1 i_{1r} + r_1 i_{1i} + x_2 i_{2r} \; .$$

From this and the Nth order complex matrix equation (23) we are led to the $2N^{th}$ order real matrix equation, the construction of which is shown in fig. 6.

4.5. Reradiated field

The field reradiated from the reflector array may be determined from the theory of antenna arrays. An antenna array is defined as a system of similar and similarly oriented antennas. A spherical coordinate system (R, θ, ϕ) with origin at O will be used for describing the reradiation pattern. The far field radiated from an antenna array is found by using the principle of pattern multiplication as the product of the field $\vec{E}_{ref}(R, \theta, \phi)$ radiated from a reference antenna (= an antenna similar and similarly oriented as the other antennas placed for example at O) and a factor $G(\theta, \phi)$ called the array characteristic, which takes into account the position of all the elements.

$$\vec{E} = \vec{E}_{ref} (R, \theta, \phi) \cdot G(\theta, \phi) \quad , \tag{33}$$

where

$$\vec{E}_{ref} = \zeta_0 I_0 \frac{e^{ikR}}{R} \; \vec{F}(\theta, \phi) \tag{34}$$

$$G = \sum_{n=1}^{N} i_n \, e^{-ik\vec{r}_n \cdot \hat{R}} \; . \tag{35}$$

Here ζ_0 is the characteristic impedance of free space, $\vec{F}$ a dimensionless vector function characteristic of the type of antennas used, $\vec{r}_n$ is the radius-vector from the point of reference to antenna n and $\hat{R}$ is a unit vector in the direction from O to the field point.

4.6. Description of the properties of the reflector

The most commonly used quantity for describing reradiation or scattering properties is the scattering cross section σ, which independent of the distance gives the far field scattered from a target for an incident plane wave. The scattering cross section in the direction opposite to the incoming wave is termed the backscattering cross section σ_b, while the cross section in an arbitrary direction is called the differential scattering cross section σ. The backscattering cross section is useful for radar applications, and was used in most of the literature mentioned in section 3.

The scattering cross section is defined (see for example Mentzer [25]) by (fig.7)

$$T(\theta,\phi) = 4\pi R^2 \frac{|\bar{S}_r(\theta,\phi) \cdot \hat{n}_r|}{|\bar{S}_i(\theta_i,\phi_i) \cdot \hat{n}_i|} \tag{36}$$

where $\bar{S}_r$ and $\bar{S}_i$ are the Poynting vectors of the reflected and incoming field, and $\hat{n}_r$ and $\hat{n}_i$ are unit vectors in the direction of the reflected and incoming field, respectively.

For the incident plane wave we have

$$|\bar{S}_i \cdot \hat{n}_i| = \frac{1}{2} \frac{E_o^2}{\zeta_o}$$

and for the reflected wave from (33) and (34)

$$|\bar{S}_r \cdot \hat{n}_r| = \frac{1}{2} \zeta_o \left(\frac{E_o \lambda}{\pi Z_o}\right)^2 \frac{F^2}{R^2} G^2 ,$$

where $F = |\bar{F}|$.

We thereby obtain

$$\sigma(\theta,\phi) = \frac{4}{\pi} \lambda^2 \left(\frac{\zeta_o}{Z_o}\right)^2 F^2 G^2 . \tag{37}$$

5. NUMERICAL EXAMPLE OF A LINEAR REFLECTOR

In order to illustrate the theory developed in the preceeings sections
a numerical investigation of a linear Van Atta array with 4 elements is induced.
The array configuration examined is shown in fig. 8, and a rectangular coordinate
system is introduced. The antenna elements are half wave dipoles ($L = \lambda/2$ paral-
lel to the y-axis and placed equidistant with separation d along the x-axis.
The transmission lines are of equal length and with a characteristic impedance
$z_o = 75 \Omega$. The dipoles are supposed to be matched to the transmission lines, so
that Z_A is 75 Ω . The primary plane wave is incident in the xz-plane and po-
larized parallel to the antennas.

The numbering of the elements is shown in the figure. The coordinates
of antenna n is

$$(x_n, y_n, z_n) = d(n - \frac{N}{2} - \frac{1}{2}, 0, 0) \tag{38}$$

and the direction of the antennas

$$(\phi_n, \theta_n) = (\frac{\pi}{2}, \frac{\pi}{2}) . \tag{39}$$

The characteristic angles of the primary wave are

$$(\phi_i, \theta_i, v) = (0, \theta_i, \frac{\pi}{2}) . \tag{40}$$

Thus we find for the normalized induced voltages from (6)

$$v_n = e^{-ikd (n - \frac{N}{2} - \frac{1}{2})} \sin \theta_i . \tag{41}$$

The self- and mutual impedances of the dipole antennas are found from
Stearns' table [23]. The factor F is for a half-wave dipole given by

$$F (\theta, \phi) = \frac{1}{2\pi} \frac{\cos(\frac{\pi}{2} \cos \phi)}{\sin \phi} = \frac{1}{2\pi} \tag{42}$$

for $\phi = \frac{\pi}{2}$ and independent of θ.

By electronic computer calculations the currents of the elements are
found from the matrix equation of fig. 6, and next the normalized scattering
cross section σ' is found from (37) and (42) giving

$$\sigma' = \frac{\sigma}{\lambda^2} = \frac{1}{\pi^3} (\frac{\zeta_o}{z_o})^2 \sigma^2 . \tag{43}$$

Reradiation patterns for various values of the parameters have been plot-
ted in figures 9 - 14.

Figures 9, 10 and 11 show the variation with the length of the transmission lines, a being $0 + p\lambda$, $\lambda/4 + p\lambda$, and $\lambda/2 + p\lambda$, respectively, p being an integer, $d = 0.2\lambda$. It is seen that there in all cases is some Van Atta effect, however maximum reradiation is usually not directly back in the direction of arrival. Further there is a normal reflector effect (mirror) giving reradiation patterns which are symmetrical with respect to the normal for $a = 0 + p\lambda$ and $\frac{\lambda}{2} + p\lambda$, (this corresponds to the two cases, where I" and I', fig. 5a, are zero, respectively). For $a = \frac{\lambda}{4} + p\lambda$ the reradiation patterns are not symmetrical, however the reflector effect is still present. Further it is seen, that for normal incidence the reradiated field is exactly zero for $a = 0 + p\lambda$ and increasing with the length of line until $a = \lambda/2 + p\lambda$.

In fig. 12 d is 0.5λ, while in fig. 10 d was 0.2λ, a in both cases being $\lambda/4 + p\lambda$. By comparison it is seen that the greater distance between adjacent elements gives rise to more, and more pronounced lobes of the reradiation pattern.

In fig. 13 is shown what happens when mutual impedances are neglected (shown for the case $a = \lambda/4 + p\lambda$ and $d = 0.2\lambda$). By comparison with fig. 10 it is shen, that the reradiation pattern becomes symmetrical and for angles of incidence around normal incidence σ' takes much greater values when mutual impedances are neglected.

Finally is in fig. 14 shown the effect of a mismatch between antennas and transmissionlines ($z_A = 73,08 - i \, 42,52\Omega$, $z_0 = 50\Omega$). It is seen that for certain values of the angle of incidence the Van Atta effect disappears completely.

The curves shown should only be taken as samples of what could be obtained with a Van Atta reflector. In the next scientific Report of the contract a more analytical treatment of a four-element linear reflector will be given. Further experimental investigations have been started. The introductory measurements agreed well with the theoretical results.

6. CONCLUSION

A method of calculation has been set up which should be applicable for investigation arbitrary Van Atta reflectors. Only linear reflectors with four elements have been treated numerically. The numerical results indicate that when the antenna elements are matched to the transmission lines a Van Atta effect is obtained to some degree. However additional effects are present, the most pronounced being, that the reflector also acts as a normal plate reflector (mirror). Further it is found that only a very small amount of energy will be reflected for angles of incidence near normal, when the transmission lines have a length which is equal to an integral number og wavelengths.

The influence of the mutual impedances and of a mismatch between antenna elements and the transmission lines indicate, that these effects might be utilized to change the reradiation pattern to compare better with a prescribed form.

The numerical results will be checked with an experimental investigation.

7. APPENDIX

In what follows a matrix equation equivalent to (23) will be set up under the simplifying assumptions made in the patent description and in all subsequent papers. These assumptions are : 1) the current used for calculating the field re-radiated from an antenna is only caused by the voltage induced at its intercon-nected mate, 2) mutual impedances are neglected.

The equivalent circuits corresponding to fig. 5 a and b will under the-se assumptions be as shown in fig. 15 a and b. From these circuits we find

$$v_m = \left[-2z_A \cos ka + i(1 + z_A^2) \sin ka\right] i_k = z' i_k \tag{44}$$

and

$$v_p \cos kl = \left[z_A \cos kl - i \sin kl\right] i_p = z'' i_p \tag{45}$$

leading to the matrix equation

$$
\begin{Bmatrix}
v_1 \\
v_2 \\
\cdot \\
v^o_{\frac{N+1}{2}} \\
\cdot \\
\cdot \\
v_N
\end{Bmatrix}
=
\begin{Bmatrix}
0 & 0 & & & & 0 & z' \\
0 & & & & & z' & 0 \\
\cdot & & & & & \cdot & \cdot \\
\cdot & & z'' & & & \cdot & \cdot \\
\cdot & & & & & \cdot & \cdot \\
0 & z' & & & & & 0 \\
z' & 0 & & & & 0 & 0
\end{Bmatrix}
\begin{Bmatrix}
i_1 \\
i_2 \\
\cdot \\
i_{\frac{N+1}{2}} \\
\cdot \\
\cdot \\
i_N
\end{Bmatrix}
, \tag{46}
$$

where

$$v^o_{\frac{N+1}{2}} = v_{\frac{N+1}{2}} \cos kl \text{ and } z'' \text{ only excists for N odd.}$$

The results of electronic computer calculations based on this equation is shown in fig. 16 for $N = 4$, $a = 0 + p\lambda$, $dr = 0.2\lambda$. It is seen that the re-flector in this case shows a pronounced Van Atta effect.

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Direction of incident
and reradiated wave

θ_i

d   d   d   d   d   d

Antenna
elements

6 kd sin θᵢ   5 kd sin θᵢ   4 kd sin θᵢ   3 kd sin θᵢ   2 kd sin θᵢ   kd sin θᵢ   0   Relative phase
of received field

0   kd sin θᵢ   2 kd sin θᵢ   3 kd sin θᵢ   4 kd sin θᵢ   5 kd sin θᵢ   6 kd sin θᵢ   Relative phase
of transmitted
field.
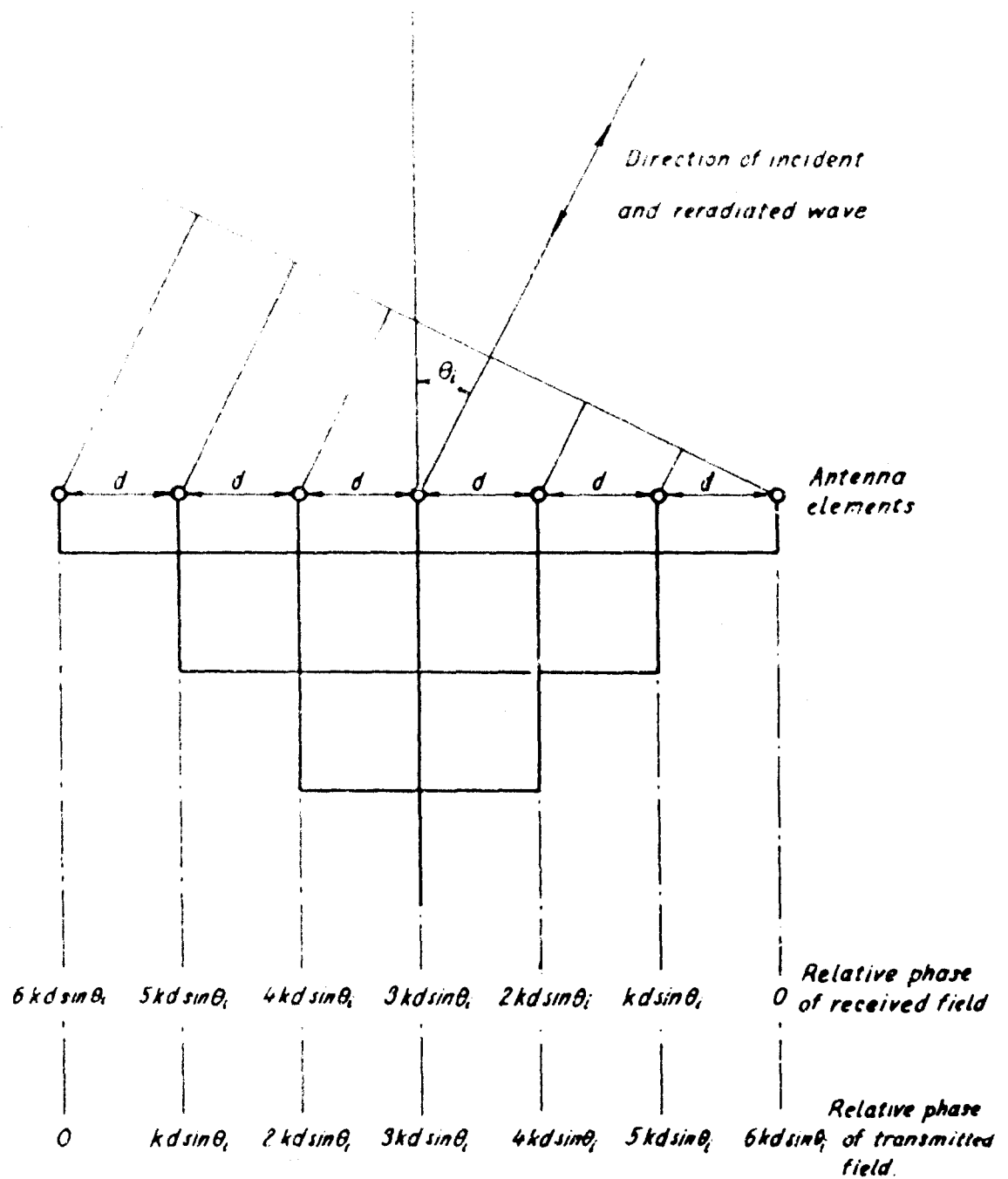
Fig 1. Simple explanation of the way of
action of a Van Atta reflector

incident plane wave

$\vec{E}_o$

plane of incidence

$\hat{z}$

θ_i

$\hat{q}$

O

φ_i

$\hat{x}$

reference plane

θ_n

φ_n

$P_n(x_o, y_o, z_o)$

antenna element
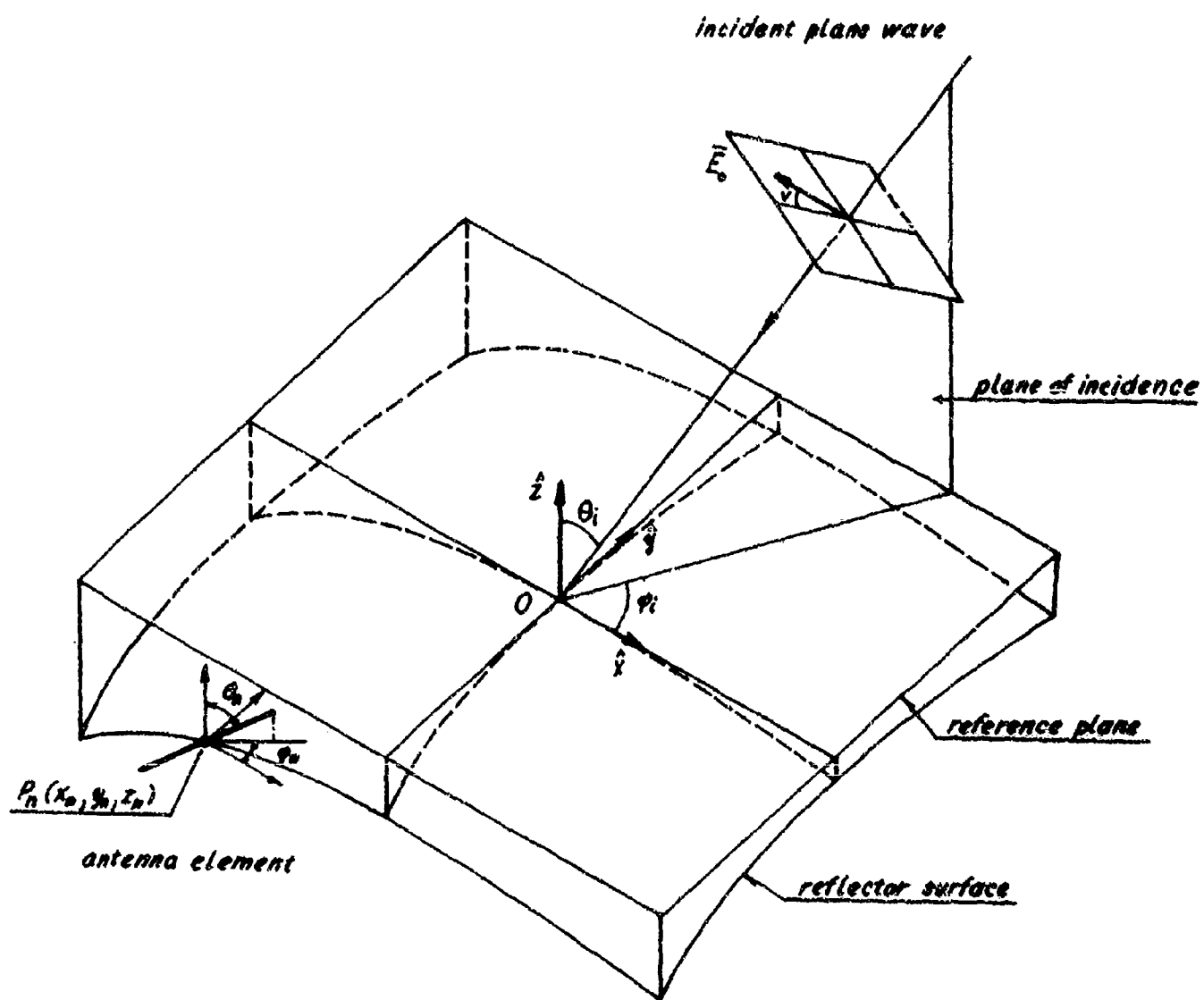
reflector surface

Fig 2. Coordinate system for reflector surface
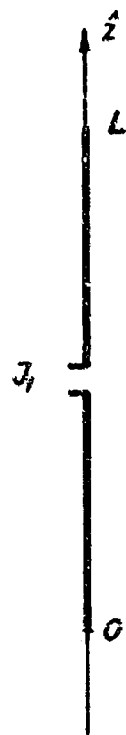
Fig 3a. Calculation of the selfimpedance of a dipole
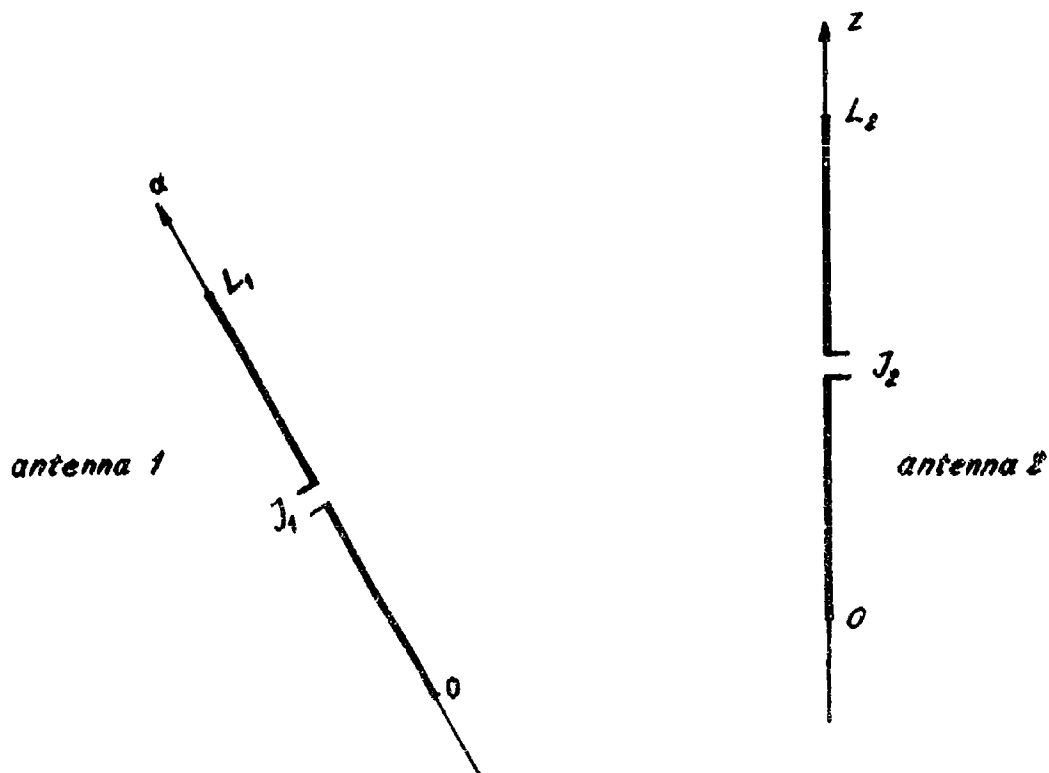
Fig 3b.   Calculation of the mutual impedance
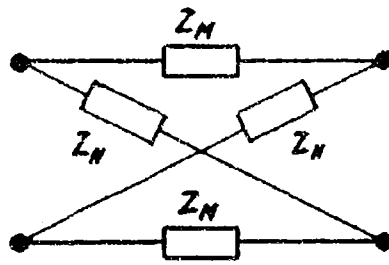
Fig 4a. Two-port X-Circuit
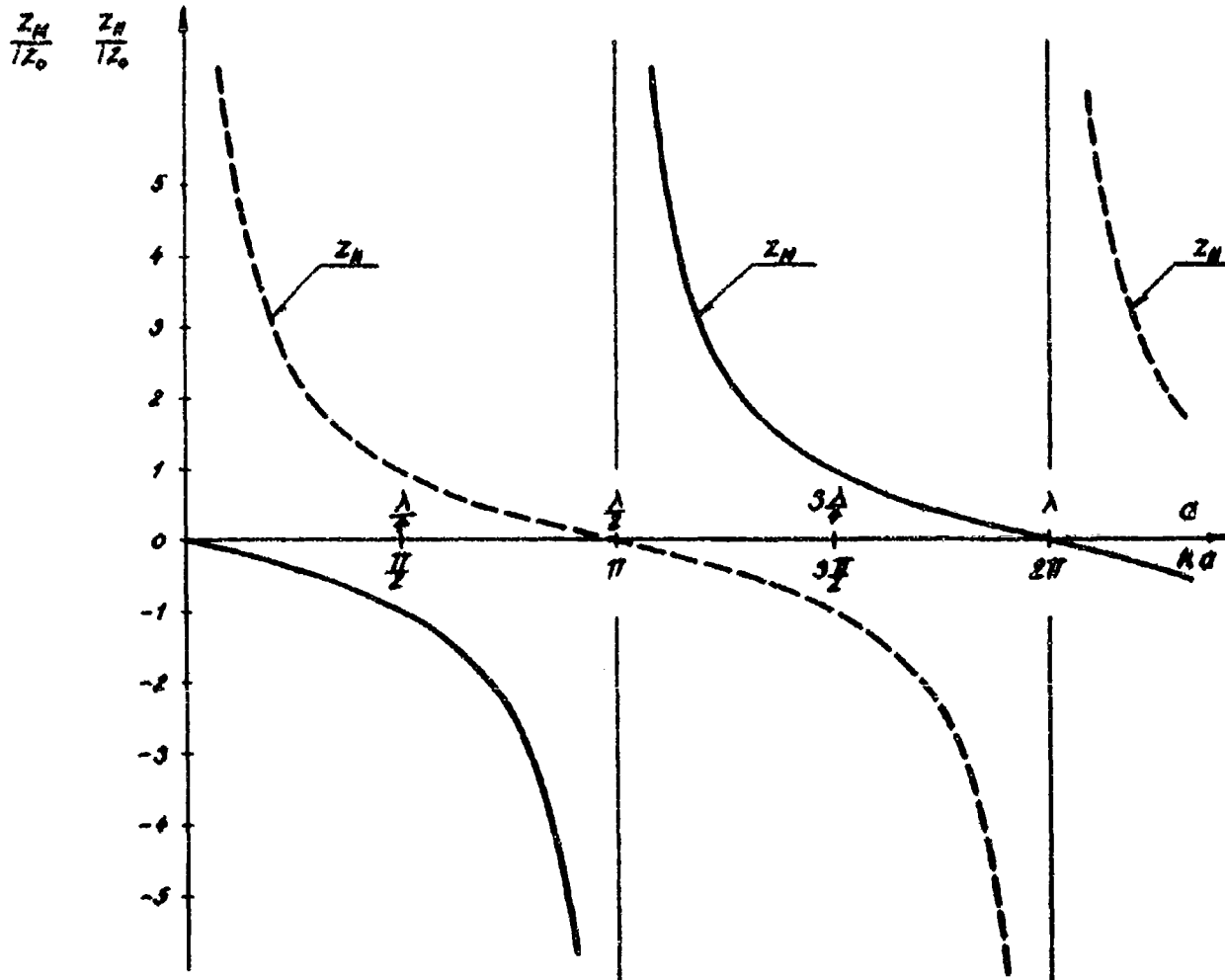
Fig 4b. Impedances as a function of length
of transmissionline

$a = (2p+1)\frac{\lambda}{2}$

$a = p\lambda$

antenna m   transmission line   antenna k

$N-1$ generators with voltages $Z_{nm} J_n$
where $1 \leq n \leq N$, $n \neq m$

$N-1$ generators with voltages $Z_{nk} J_n$
where $1 \leq n \leq N$, $n \neq k$

Fig 5a. Equivalent circuit of two interconnected array elements

antenna p   transmission line

$N-1$ generators with voltages $Z_{np} J_n$
where $1 \leq n \leq N$, $n \neq p$

**Fig 5b. Equivalent circuit of array element connected
to short-circuited transmission line**

-, O and + refer to superscripts of the voltages (25), (26) and (31).

The boxes of the upper half of the matrix should be filled like $\boxed{r \quad -x}$

"   "   "   " lower   "   "   "   "   "   "   "   " $\boxed{x \quad r}$

where r and x refer to real and imaginary parts of the impedances.

Signature used for the various impedances :

$\blacksquare$ z_{ik}^{-} formula (27)

$\square$ z_{im}^{-} formula (28)

$\equiv$ z_{ik}^{+} formula (29)

$\boxed{\bullet}$ z_{in}^{+} formula (30)

$\boxed{+}$ $z_{S+1,S+1}^{0}$ formula (32)

$\diagup$ $z_{S+1,m}^{0}$ formula (33)

The double arrows indicate same value and opposite sign.

" single " " " " " " same "

R and I indicate real and imaginary parts of the currents respectively.

Fig 6. Construction of matrix equation with real elements
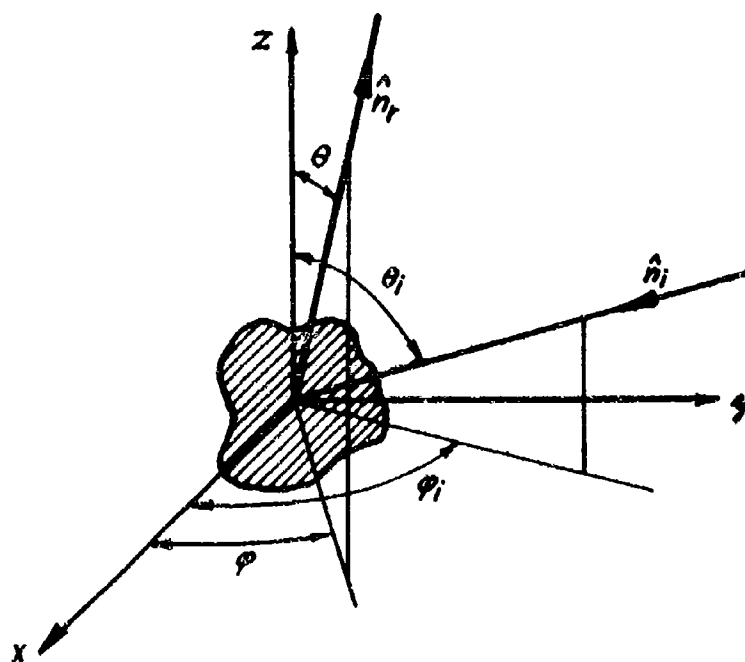
Fig 7. Definition of scattering cross section
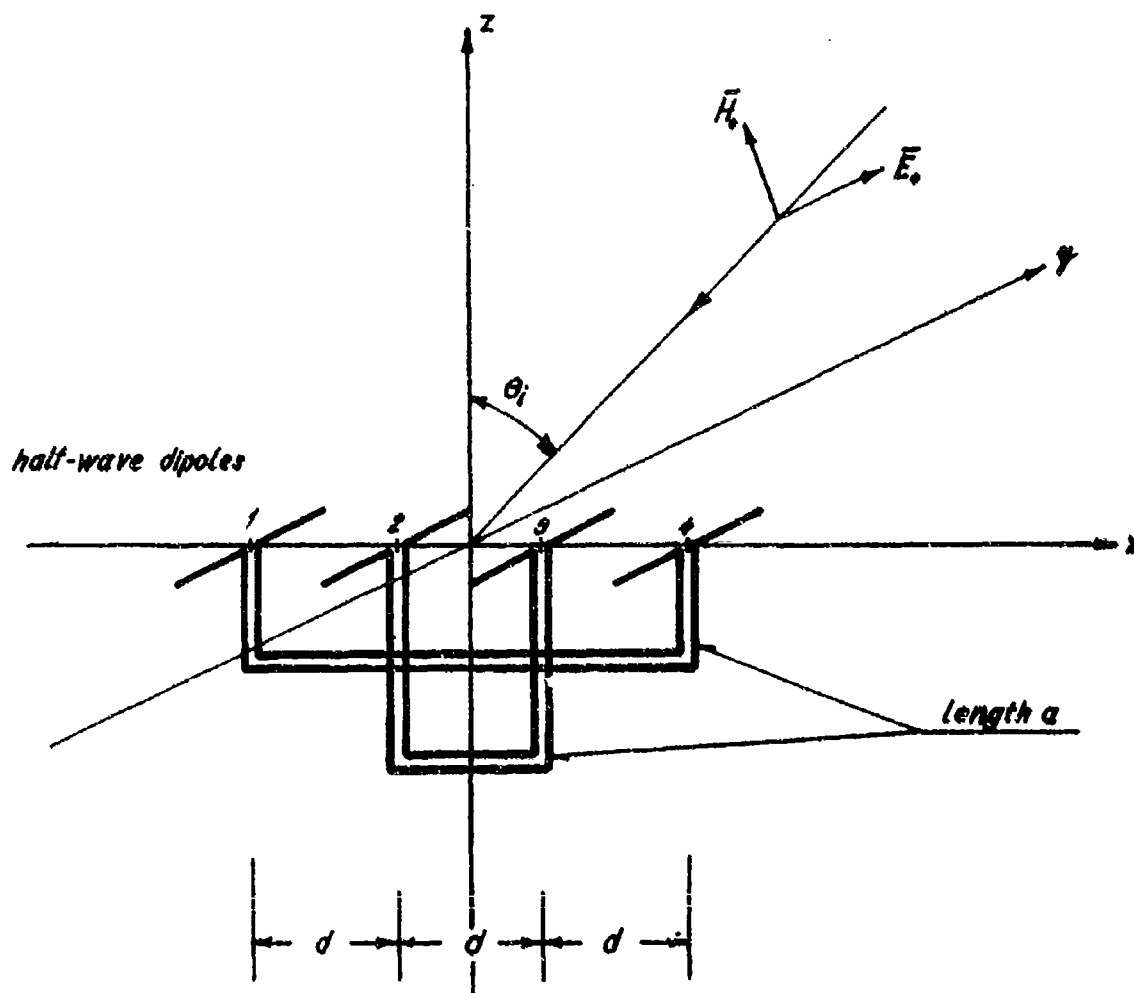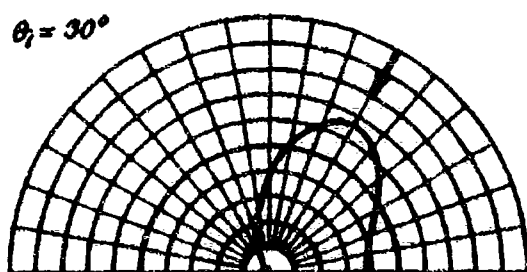
Fig 8. Reflector array investigated numerically
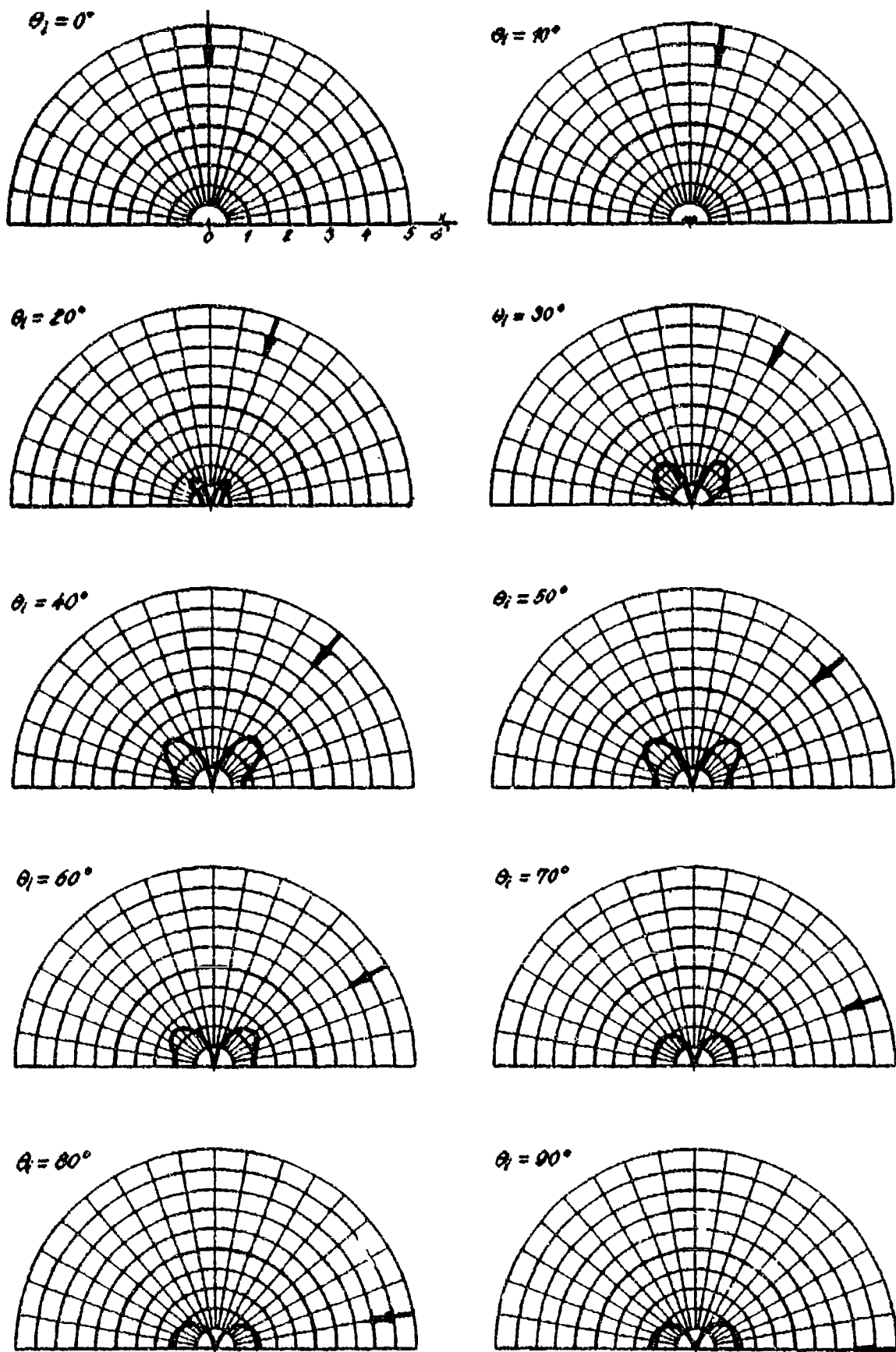
Fig 9. Relative differential scattering cross section σ'
as a function of the angle of incidence. $a = 0 + p\lambda$,
$d = 0.2\lambda$, $Z_e = Z_A = 75\Omega$.
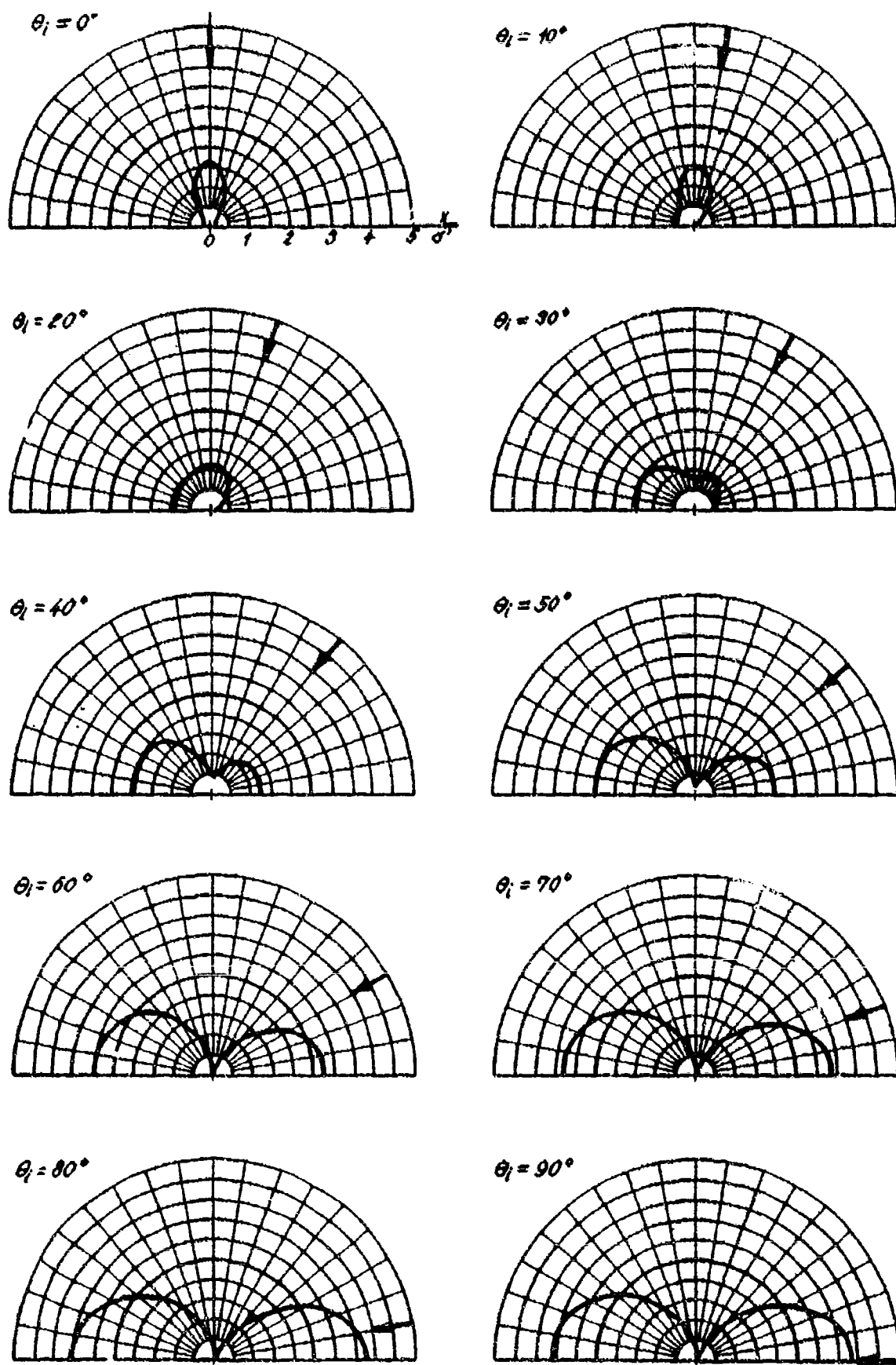
Fig 10. Relative differential scattering cross section σ' as a function of the angle of incidence. $a = \lambda/4 + p\lambda$, $d = 0.2\lambda$, $Z_0 = Z_A = 75\,\Omega$.

Fig 11. Relative differential scattering cross section σ'
as a function of the angle of incidence. $a = \lambda/8 + p\lambda$,
$d = 0.2\,\lambda$, $Z_o = Z_A = 75\,\Omega$.

$\theta_i = 10°$

$\theta_i = 20°$ $\theta_i = 30°$

$\theta_i = 40°$ $\theta_i = 50°$

$\theta_i = 60°$ $\theta_i = 70°$

$\theta_i = 80°$ $\theta_i = 90°$
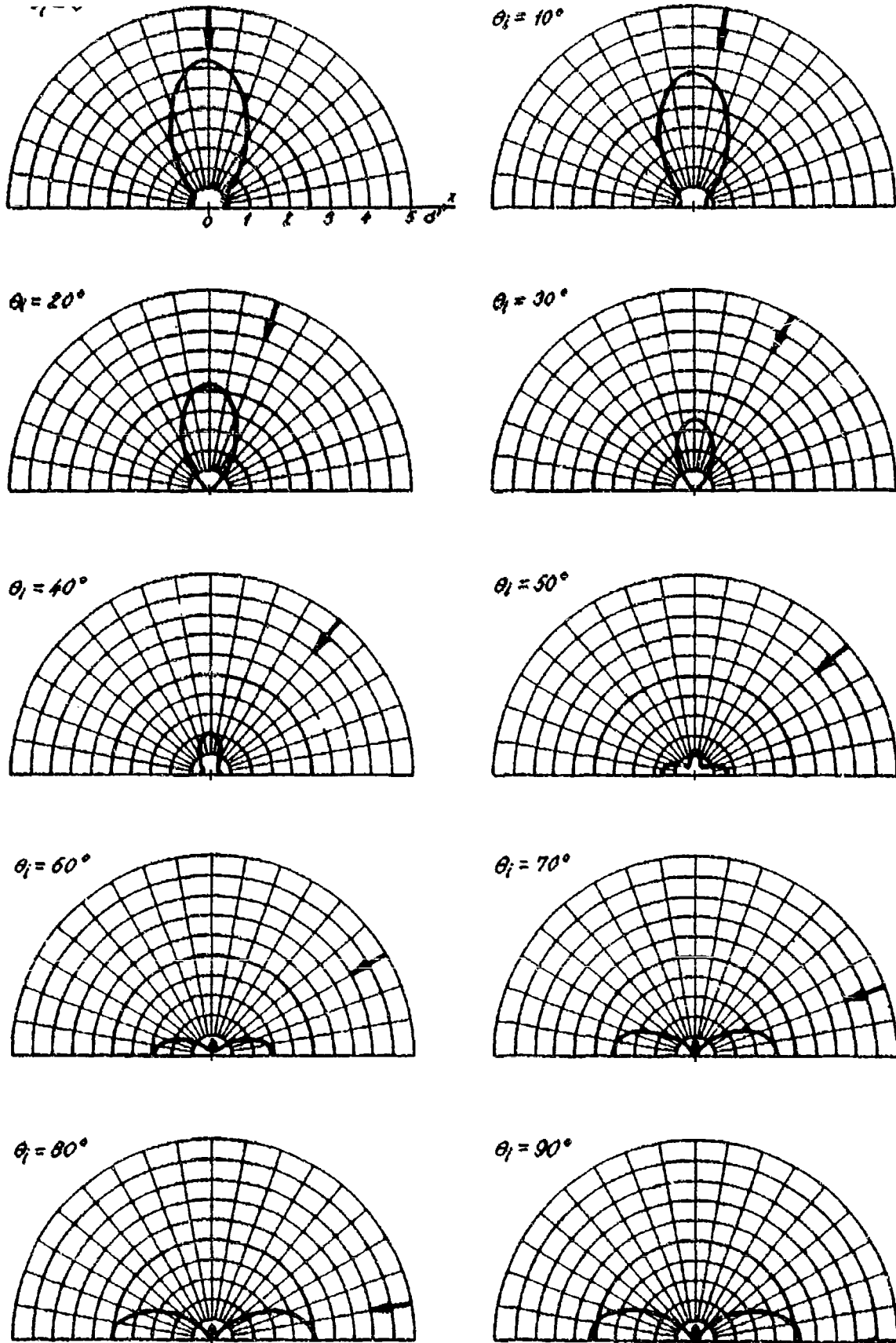
Fig 12. Relative differential scattering cross section σ' as a function of the angle of incidence. $a = \lambda/4 + p\lambda$, $d = 0.5\lambda$, $Z_o = Z_p = 73.08\,\Omega$.
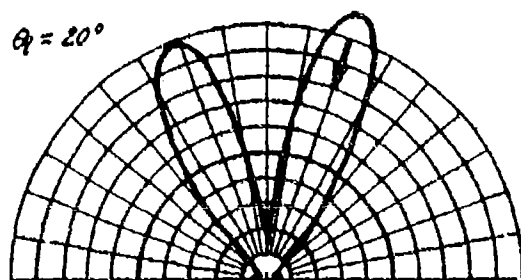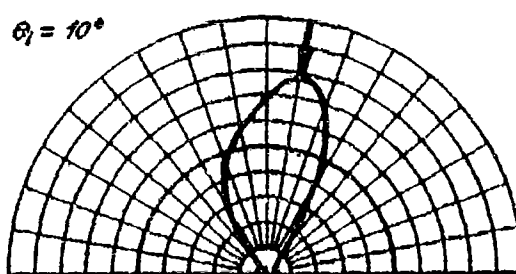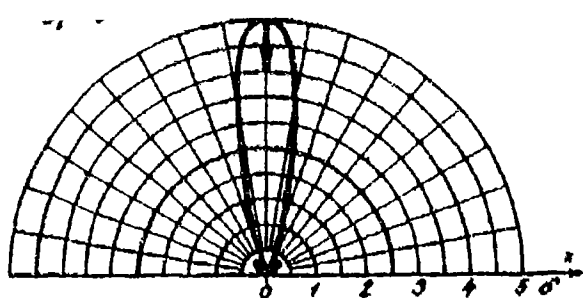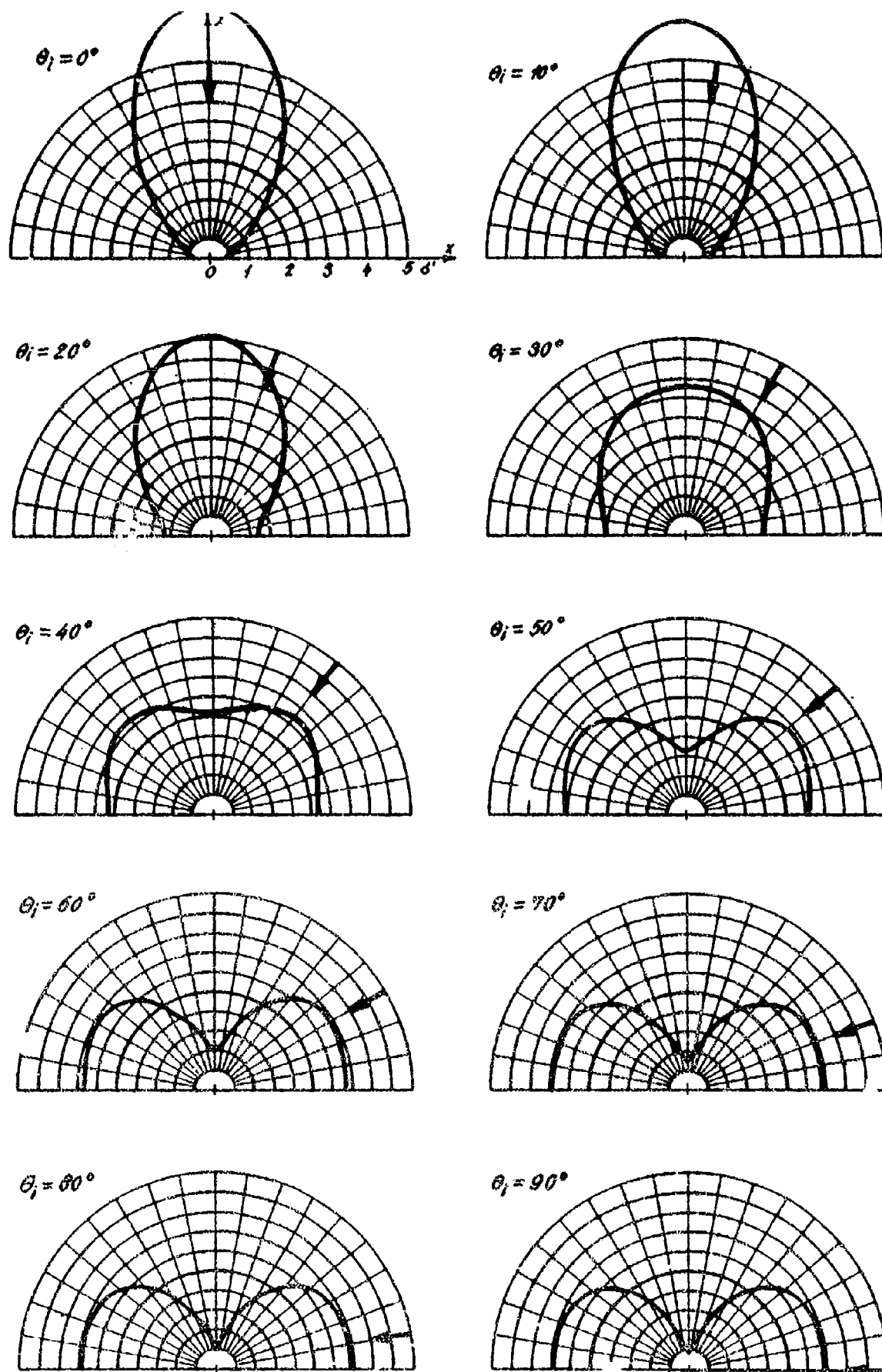
Fig.13. Relative differential scattering cross section σ'
as a function of the angle of incidence. $a = \lambda/4 + p\lambda$,
$d = 0.2\lambda$, $Z_o = Z_L = 75\,\Omega$. Mutual impedances zero.
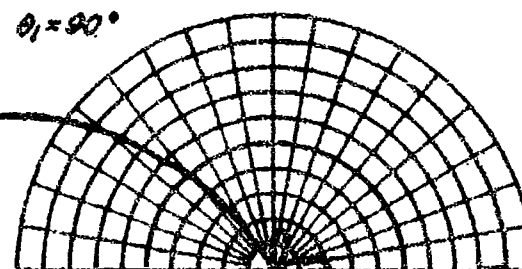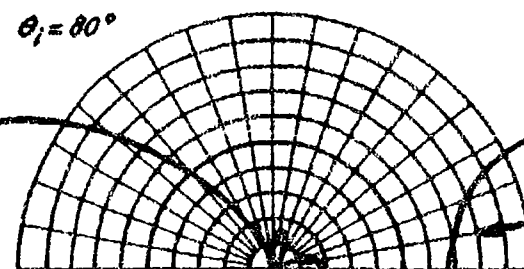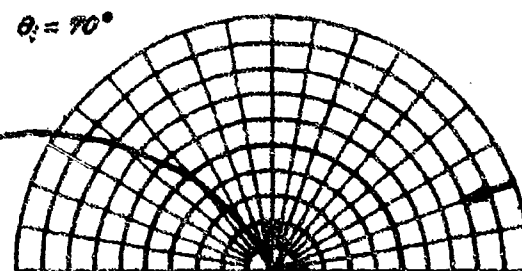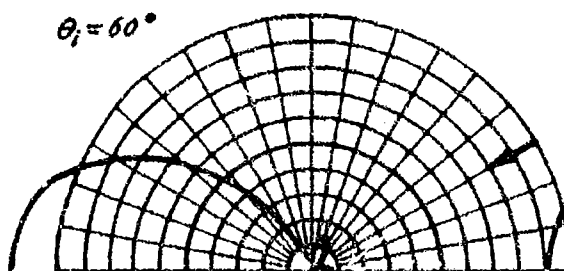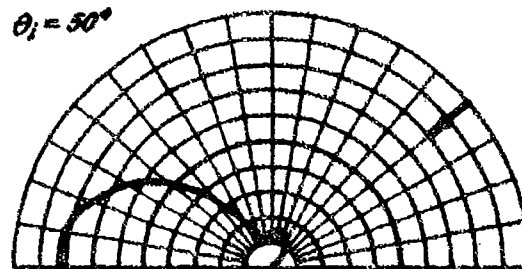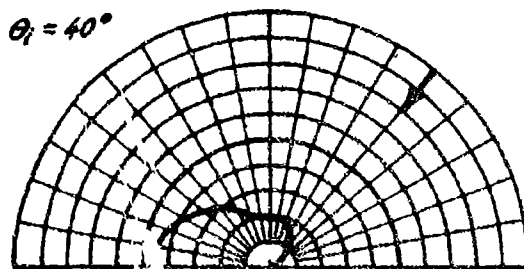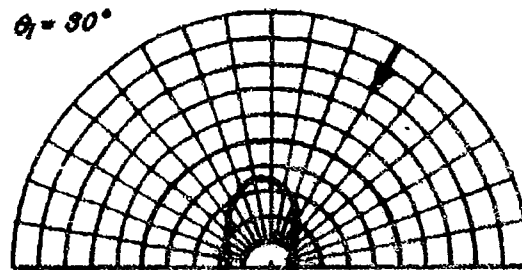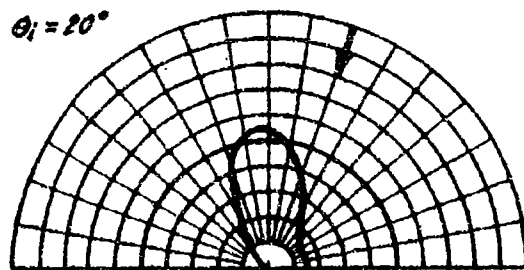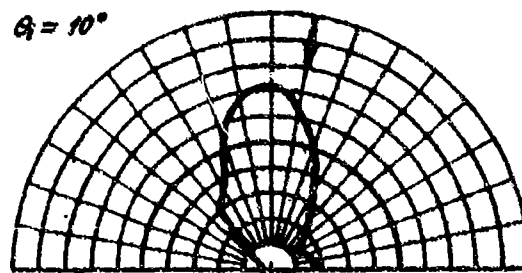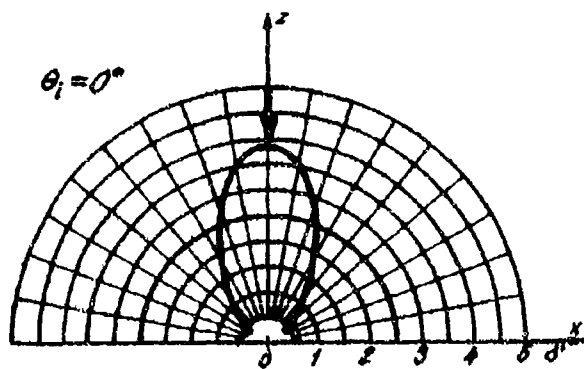
Fig 14. Relative differential scattering cross section σ'
as a function of the angle of incidence. a = λ/₂ + pλ,
d = 0.2 λ, Z = 50 Ω, Z = 73.08 − i 42.52 Ω
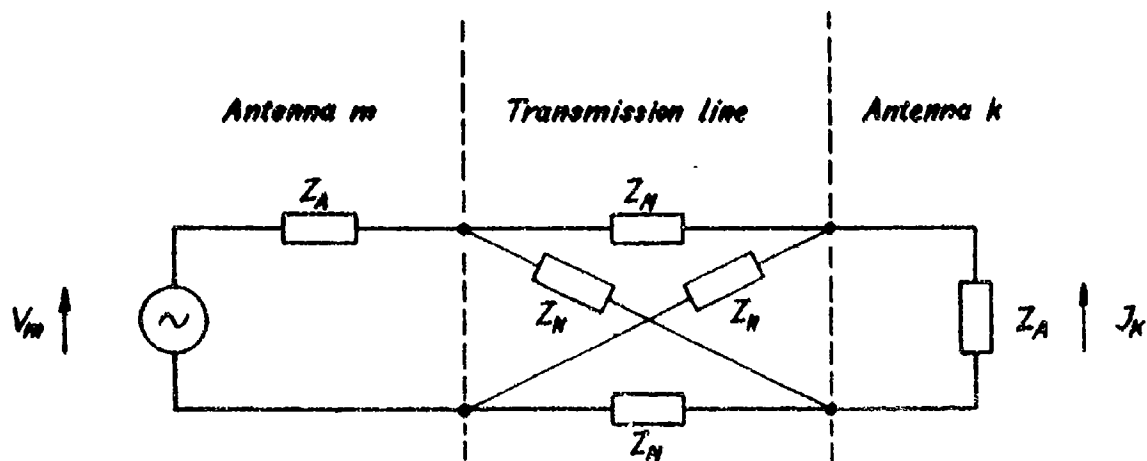
Fig 15a. Equivalent circuit for calculating the current in an antenna due to the voltage at its mate according to the the patent description.
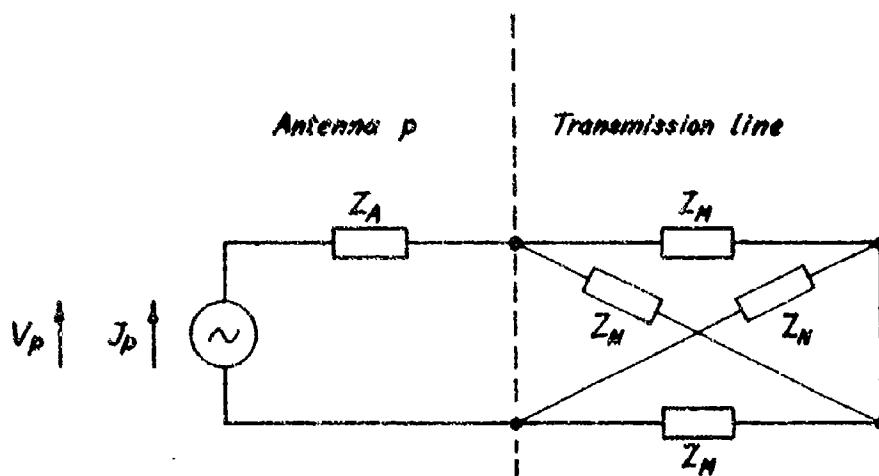
Fig 15b. Equivalent circuit for an antenna connected to a short circuited transmission line according to the patent description.
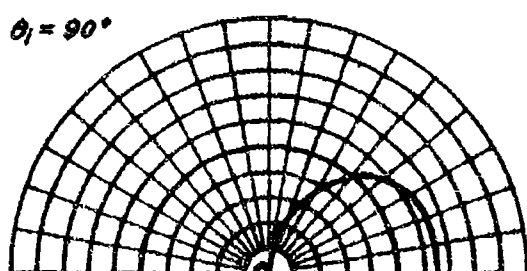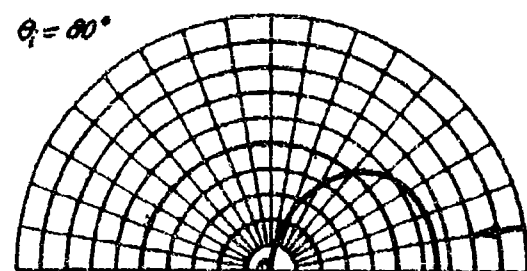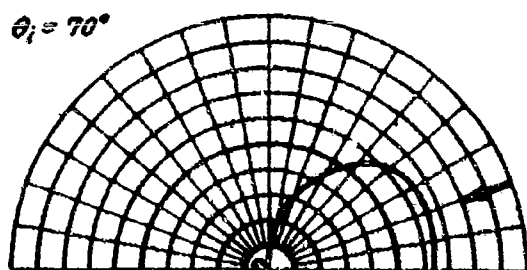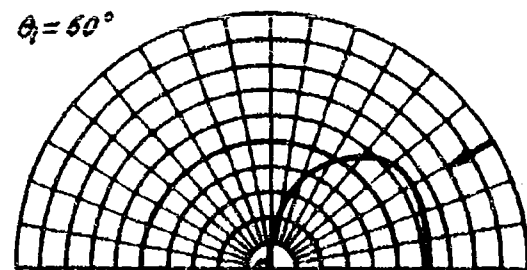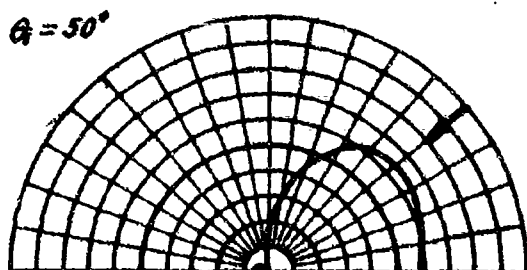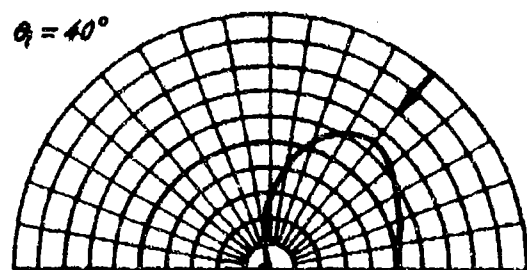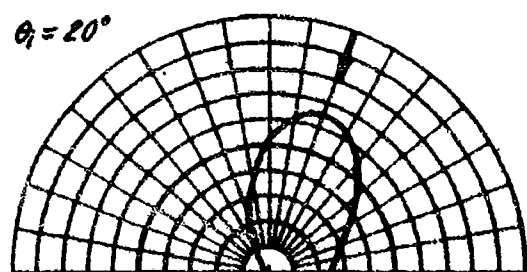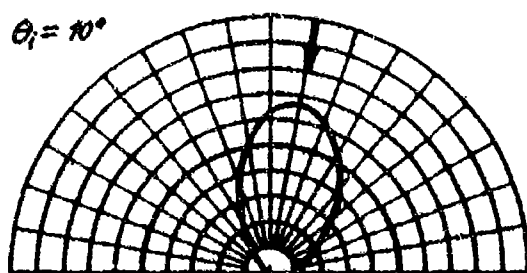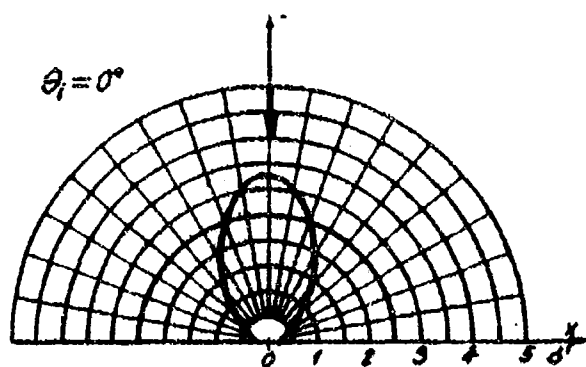
Fig 16. Relative differential scattering cross section σ'
as a function of the angle of incidence calculated
in accordance with Van Atta's patent description.

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LYNGBY, DENMARK
E-EF REPORT NO. R 39
NOVEMBER 1964

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SR 1
ELECTRONICS

A THEORETICAL INVESTIGATION OF VAN ATTA ARRAYS

TOVE LARSEN

ABSTRACT: A scheme has been set up for an analytical investigation of an arbitrary Van Atta reflector. Each pair of antenna elements is represented by an equivalent circuit using equivalent x-circuits for the transmissionlines. Reradiation from the elements and mutual impedances have been taken into account. The theory is illustrated with some numerical examples of a linear reflector consisting of four half-wave dipoles.